Minimum total potential energy principle

I. Strain energy due to torsion :

$$U_T = \frac{1}{2}\Phi T = \frac{1}{2}\Phi^2 \frac{GJ}{L}$$

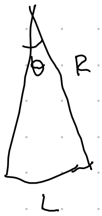
G: shear modulus

J: torsion constant

Φ: deformation relative to minimal twist angle.

II. Strain energy due to bending

$$U_B = \frac{1}{2}\Theta M = \frac{1}{2}\frac{LEI}{R^2}$$



E: Young's modulus

I: moment of inertia

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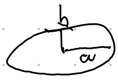
for ellipse :



$$J \approx \frac{\pi a^3 b^3}{a^2+b^2}$$

$E = 3890$ MPa
$G = 890$ MPa
} Table 9.5 in Chemical and Physical Behavior of Human Hair

$$I = \frac{\pi ab}{4}(a^2\sin^2\theta + b^2\cos^2\theta)$$, θ is the angle relative to curvature vector

$$= \frac{\pi ab^3}{4} + \frac{\pi ab}{4}(a^2-b^2)\sin^2\theta$$

$$\frac{1}{R} = \frac{\|r' \times r''\|}{\|r'\|^3} \qquad L = \|r'\|\int dt$$

To minimize:

$$U = W_t(\theta_t - \theta)^2 + W_b\sin^2\theta \quad , \theta = [0, \frac{\pi}{2}]$$

θ_t is the angle between the curvature vector and the minimal twist normal.

$$\frac{\partial U}{\partial \theta} = 2W_t(\theta - \theta_t) + W_b\sin 2\theta$$

$$\frac{\partial^2 U}{\partial \theta^2} = 2W_t + 2W_b\cos 2\theta$$